Principles of Measurement Systems

Exam - PoMS, 07/11/2011

- Write each question on a sheet of paper.
- Write your **name** and **student ID** on each sheet.
- Pay attention to units. A numerical result without a unit will be considered wrong!
- Only a regular calculator is allowed.
- This is NOT an open book exam.
- You are allowed to bring one A4 page with your own notes (one side only).
- You have **3 hours** to complete the exam.

Question 1: General (2.5 points)

$\frac{1}{2}$ a) Explain the working principle of a Hall sensor and what it can measure.

$\frac{1}{2}$ b) What is the Nyquist sampling theorem and explain its underlying principle.

$\frac{1}{2}$ c) What is reluctance and explain qualitatively the working principle of a variable reluctance tachogenerator.

$\frac{1}{2}$ d) What is the working principle behind Amplitude Modulation (AM) and explain how such a technique can help to reduce external interferences?

$\frac{1}{2}$ e) Explain qualitatively the working principle of one possible implementation of an Analogue to Digital Converter (ADC).

Table 1: Input for exam question 2.

	Pitot tube	Differential pressure transmitter	ADC	Microcontroller with display
model equations	$\Delta P = \frac{1}{2}\rho v_T{}^2$	$i = K_1 \Delta P + a_1$	$n = K_2 i + a_2$	$v_M = K_3\sqrt{n^{1.01} - 59}$
mean values	$\overline{\rho} = 1.3$	$\overline{K_1} = 0.064$ $\overline{a_1} = 4.0$	$\overline{K_2} = 12.2$ $\overline{a_2} = 0.0$ with n being rounded off to nearest integer	$\overline{K_3} = 1.44$
Half-widths of rectangular distribution	$h_\rho = 0.12$	$h_{a_1} = 0.04$	$h_{a_2} = 0.5$	$h_{K_3} = 0.0$

1

Question 2: A fluid velocity measurement system (1.5 points)

A fluid velocity measurement system consists of a pitot tube, a differential pressure transmitter, an analogue-to-digital converter (ADC) and a microcontroller with display facilities. Table 1 gives the model equations and parameters for each element in the system. The microcontroller calculates the measured value of velocity assuming a constant density.

a) Show that for a rectangular error distribution with a half width of h, the standard deviation $\sigma = h/\sqrt{3}$.

b) Estimate the mean and standard deviation of the error probability density function assuming the true value of velocity, v_T, to be 14 m/s.

Question 3: A force measurement system (1.5 points)

A force measurement system consists of linear elements and has an overall steady-state sensitivity of unity. The dynamics of the system are determined by the second-order transfer function of the sensing element, which has a natural frequency $\omega_n = 35$ rad/s and a damping ratio $\xi = 0.15$. Calculate the system dynamic error corresponding to the periodic input force signal:

$$F(t) = 50\,(\sin 10t + 1/3 \sin 30t + 1/5 \sin 50t)\,,$$

with t in seconds and $F(t)$ in Newtons.

Question 4: A potentiometer (2 points)

A potentiometer has a total length of 10 cm and a resistance of 200 Ω.

a) Calculate the supply voltage so that the power dissipation is 1 W.

b) Draw the Thévenin equivalent circuit for an 8 cm displacement.

c) The potentiometer is connected to a recorder with a resistance R_L. Find R_L such that the recorder voltage is 7% less than the open circuit voltage at an 8 cm displacement.

2

Question 5: A strain gauge measurement system (2.5 points)

Consider a strain gauge measurement system as indicated in Fig. 1. The sensor consists of 4 strain gauges for which R_1 and R_4 are placed in tensile mode, e.g. $R_1 = R_4 = R_0(1+Ge)$, and R_2 and R_3 in compressive mode, e.g. $R_2 = R_3 = R_0(1-Ge)$, with $R_0 = 100\ \Omega$ at a temperature of $T = 20\ °\text{C}$ and a gauge factor of $G = 2$. The power supply has a voltage of $V_S = 12$ V. The sensor is connected to a recorder element via a cable with a total resistance of $R_C = 50\ \Omega$. The recorder has a loading resistance of $R_L = 10$ kΩ.

$\frac{1}{2}$ a) Find the Thévenin equivalent voltage, V_{Th}, and the corresponding impedance, Z_{Th}, of the sensor element.

$\frac{1}{2}$ b) Calculate the voltage over R_L, V_L, for a strain of $e = 10^{-3}$ at $T = 20\ °\text{C}$. How large is the loading effect on the recorder?

1 c) The temperature of the sensor increases, which leads to an increase in the resistances of the strain elements, R_1, R_2, R_3, and R_4, by 5 Ω. What is the main underlying physical mechanism that increases the resistance of a conductor due to an increase in temperature? Discuss the influence of this effect on V_L.

$\frac{1}{2}$ d) The output voltage on the recorder, V_L, is considered to be too small. Redesign the measurement system in such a way that V_L is amplified by a factor 10. For this, introduce an ideal operational amplification element without changing the sensor and recorder elements and their parameters.

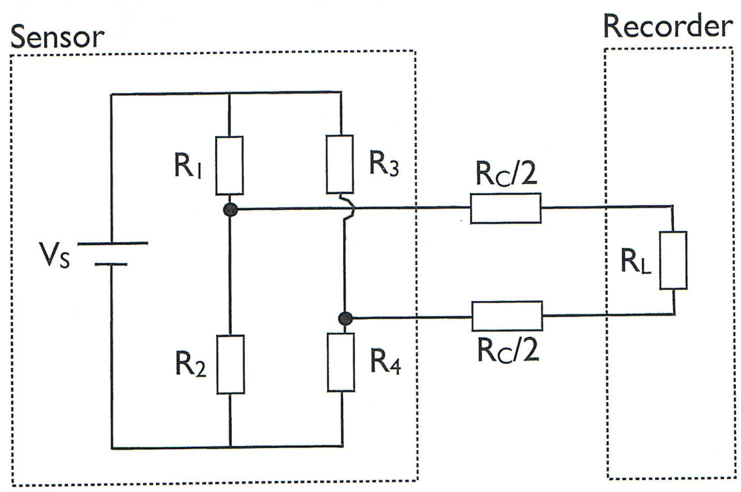


Figure 1: Figure corresponding to question 5.

3

see book

(a) 8.10; p196-197

(b) 10.1.1; p247-249

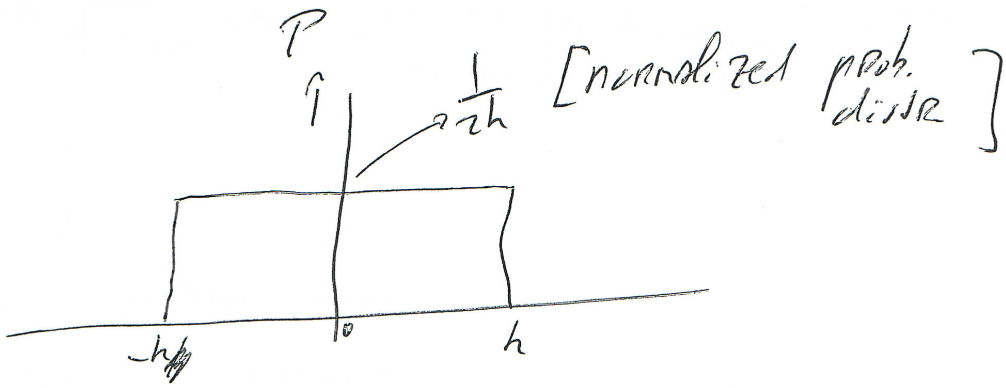
(c) 8.3.1; p165; 8.4; p170-172

(d) 9.3; p224-227

(e) 10.1.6; p256-260

2)

a)



$$\sigma^2 = \int_{-h}^{h} (x - \underbrace{\bar{x}}_{=0})^2 P(x)\,dx = \frac{1}{2h}\int_{-h}^{h} x^2\,dx$$

$$= \frac{1}{2h}\left[\frac{1}{3}x^3\Big|_{-h}^{h}\right] = \frac{1}{2h}\left[\frac{1}{3}h^3 + \frac{1}{3}h^3\right]$$

$$= \frac{h^2}{3} \Rightarrow \underline{\underline{\sigma = \frac{h}{\sqrt{3}}}}$$

$\frac{1}{2}$

Question 2

b)

$$I_1 = v_T \longrightarrow \boxed{\text{Pitot}} \xrightarrow{O_1 = I_2} \boxed{\text{Differential Pressure Transmitter}} \xrightarrow{O_2 = I_3} \boxed{\text{ADC}} \xrightarrow{O_3 = I_4} \boxed{\text{Micro-controller}} \xrightarrow{O = v_M}$$

$$v_T = 14 \left(\frac{m}{s}\right)$$

$$\bar{E} = \bar{O} - \bar{I}_1 = \bar{O} - 14$$

$$\bar{O}_1 = \frac{1}{2}\bar{\rho}\,\bar{v}_T^2 = \frac{1}{2} \times 1.3 \times (14)^2 = 127.400$$

$$\bar{O}_2 = \bar{K}_1 \bar{O}_1 + \bar{a}_1 = 0.064 \times 127.4 + 4 = 12.154$$

$$\bar{O}_3 = \bar{K}_2 \bar{O}_2 + \cancelto{0}{\bar{a}_2} = 12.2 \times 12.154 = 148.28 \xrightarrow{\text{round off}} \bar{O}_3 = 148$$

$$\bar{O} = \bar{K}_3 \sqrt{(\bar{O}_3)^{1.01} - 59} = 1.44\sqrt{(148)^{1.01} - 59} = 14.152$$

$$\Rightarrow \boxed{\bar{E} \approx 0.15 \left(\frac{m}{s}\right)}$$

$$\sigma_{I_1}^2 \equiv \sigma_{v_T}^2 = 0$$

$$\sigma_{I_2}^2 \equiv \sigma_{O_1}^2 = \left(\frac{\partial \Delta P}{\partial \rho}\right)^2 \sigma_\rho^2 = \left(\frac{1}{2} v_T^2\right)^2 \times \left(\frac{0.12}{\sqrt{3}}\right)^2 = 46.099$$

$$\sigma_{I_3}^2 \equiv \sigma_{O_2}^2 = \left(\frac{\partial i}{\partial O_1}\right)^2 \sigma_{O_1}^2 + \left(\frac{\partial i}{\partial a_1}\right)^2 \sigma_{a_1}^2 = K_1^2 \times 46.099 + 1 \times \left(\frac{0.04}{\sqrt{3}}\right)^2 = 0.189$$

$$\sigma_{I_4}^2 \equiv \sigma_{O_3}^2 = \left(\frac{\partial n}{\partial O_2}\right)^2 \sigma_{O_2}^2 + \left(\frac{\partial n}{\partial a_2}\right)^2 \sigma_{a_2}^2 = K_2^2 \times 0.189 + 1 \times \left(\frac{0.5}{\sqrt{3}}\right)^2 = 28.214$$

$$\sigma_O^2 \equiv \sigma_{v_M}^2 = \left(\frac{\partial v_M}{\partial O_3}\right)^2 \sigma_{O_3}^2 + \cancelto{0}{\left(\frac{\partial v_M}{\partial k_3}\right)^2 \sigma_{k_3}^2}$$

$$= \left(\frac{k_3}{2}\left(n^{1.01} - 59\right)^{-\frac{1}{2}} \cdot (1.01) \cdot n^{0.01}\right)^2 \times 28.214 = 0.171 \qquad \text{with } n = 148$$

$$\Rightarrow \boxed{\sigma_O \approx 0.41 \left(\frac{m}{s}\right)}$$

1

Steady-state sensitivity of 1 : $\frac{1}{k} = 1 \left(\frac{m}{N}\right)$

natural frequency : $\omega_n = 35 \left(\frac{rad}{s}\right)$

damping ratio : $\xi = 0.15$

$$E(t) = F_{out}(t) - F_{in}(t)$$

From page 63 of Bentley : $F_{out}^{(n)}(t) = \hat{I}^{(n)} \cdot (K) \cdot |G(j\omega_n)| \cdot \sin(\omega_n t + \phi_n)$ $\xrightarrow{} 1$

$$\Rightarrow F_{out}(t) = 50\left[|G(10j)| \sin(10t+\phi_{10}) + \frac{|G(30j)|}{3} \sin(30t+\phi_{30}) + \frac{|G(50j)|}{5} \sin(50t+\phi_{50})\right.$$

Second-order transfer function :

$$\begin{cases} G(s) = \dfrac{1}{\frac{1}{\omega_n^2} s^2 + \frac{2\xi}{\omega_n} s + 1} \\ |G(j\omega)| = \dfrac{1}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + 4\xi^2 \left(\frac{\omega}{\omega_n}\right)^2}} \end{cases}$$

$$\Rightarrow |G(10j)| = \frac{1}{\sqrt{\left(1-\frac{10^2}{35^2}\right)^2 + 4 \times 0.15^2 \times \left(\frac{10}{35}\right)^2}} = 1.08$$

$$\& \ \phi_{10} = \arg G(j\omega)\Big|_{\omega = 10\left(\frac{rad}{s}\right)} = -\arctan\left[\frac{2\xi \frac{\omega}{\omega_n}}{1-\frac{\omega^2}{\omega_n^2}}\right]_{\omega=10\left(\frac{rad}{s}\right)} = -5.33^\circ$$

$-0,093$

Finally, we will have : $|G(30j)| = 2.71$, $|G(50j)| = 0.89$

, $\phi_{30} = -44.1^\circ$, $\phi_{50} = -157.6^\circ$

$-0,77$ $+0,39$

Thus,

$$E(t) = 50\left[1.08 \sin(10t - 5^\circ) - \sin(10t)\right] + \frac{50}{3}\left[2.71 \sin(30t - 44^\circ) - \sin(30t)\right]$$
$$+ \frac{50}{5}\left[0.89 \sin(50t - 158^\circ) - \sin(50t)\right]$$

$1\frac{1}{2}$

1) $L = 10\,(cm)$, $R_p = 200\,(\Omega)$ (See Fig 5.6 of Bentley)

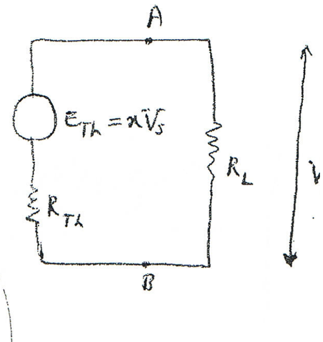
a) $\frac{V_s^2}{R_p} = \overset{Watt}{1} \Rightarrow V_s = \sqrt{R_p} = 10\sqrt{2}$ (Volts) $\frac{1}{2}$

b)

$x = \frac{8}{L} = 0.8$

$E_{Th} = V_s \cdot x$: Eq. (5.8)

$= 10\sqrt{2} \times 0.8 = 8\sqrt{2}$ (Volts)



$\frac{1}{2}$

$R_{Th} = R_p \cdot x(1-x)$: Eq. (5.9)

$= 200 \times 0.8 \times 0.2 = 32\,(\Omega)$

c) $V_L = 0.93\, E_{Th}$ at $x = 0.8 \longrightarrow R_L = ?$

$V_L = E_{Th} \frac{R_L}{R_{Th} + R_L}$: Eq. (5.2)

1

$\Rightarrow 0.93 = \frac{R_L}{32 + R_L} \Rightarrow R_L = 425\,(\Omega)$

$= \frac{32\,\Omega}{1.075 - 1} = 425\,\Omega$

$V_L = E_{Th} \frac{R_L}{Z_T + R_L}$

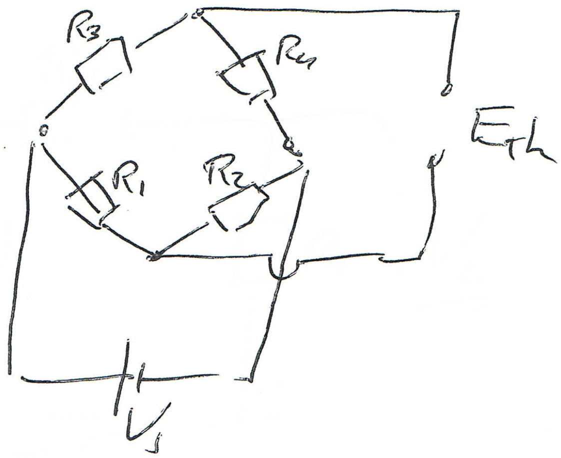
$R_L = \frac{Z_T}{\frac{E_{Th}}{V_L}\left(1 - \frac{V_L}{E_T}\right)} = \frac{Z_T}{\frac{E_{Th}}{V_L}}$

$\frac{V_L}{E_{Th}}\left(Z_T + R_L\right) = R_L$

$\frac{V_L Z_T}{E_{Th}} = R_L\left(1 - \frac{V_L}{E_T}\right) \Rightarrow R_L = \frac{V_L Z_T}{E_{Th}\left(1 - \frac{V_L}{E_T}\right)}$

a)

sensor = (wheatstone bridge)



$$E_{Th} = V_s\left[\frac{R_1R_4 - R_2R_3}{(R_1+R_2)(R_3+R_4)}\right] = V_s\left[\frac{R_0^2(1+Ge)^2 - R_0^2(1-Ge)^2}{4R_0^2}\right]$$

$$= V_s\left[\frac{4Ge}{\cancel{4}}\right] = \underline{\underline{V_s\,Ge}}$$

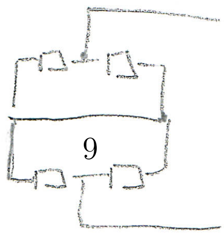
$$Z_{Th} = \frac{R_3R_4}{R_3+R_4} + \frac{R_1R_2}{R_1+R_2} = \frac{R_0^2(1-Ge)(1+Ge)}{2R_0} + \frac{R_0^2(1+Ge)(1-}{2R_0}$$

$$= R_0(1-Ge)(1+Ge) = \underline{\underline{R_0\left[1-(Ge)^2\right]}}$$

1/2

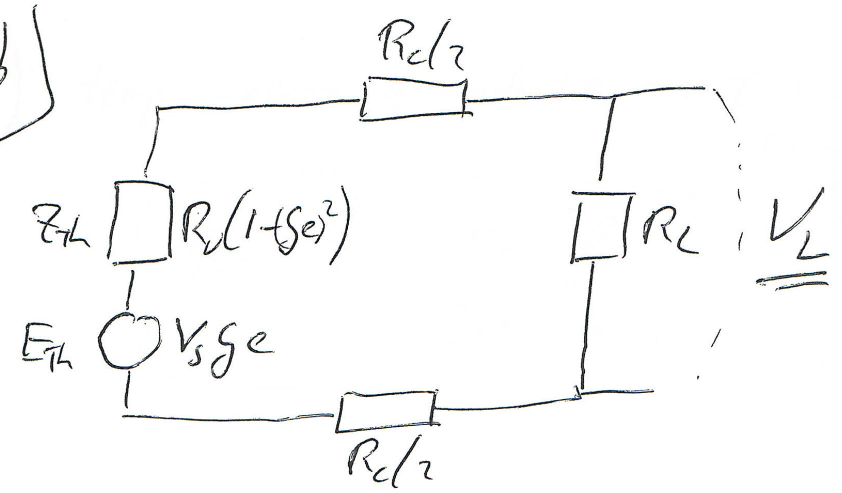
$$\frac{R_2}{R_2+R_1} - \frac{R_4}{R_3+R_4}$$

$$= \frac{R_2(R_3+R_4) - R_4(R_2+R_1)}{(R_1+R_2)(R_3+R_4)}$$



$$E_{Th} = \frac{R_1}{R_1+R_2} - \frac{R_3}{R_3+R_4}$$

b)



$$V_L = E_{Th}\,\frac{R_L}{R_L+R_c+Z_{Th}} = V_s\, g\, e\, \frac{10\cdot 10^3\ \Omega}{10\cdot 10^3+50+100\ \Omega}$$

$$= V_s \cdot g \cdot e \cdot 0{,}985 = \underline{\underline{23.6\ \text{mV}}}$$

<u>loading effect</u>: $1-0{,}985 = \underline{\underline{0.015}} = \underline{\underline{1{,}5\%}}$

$\frac{1}{2}$

(c) temp. increase of conductor $\Rightarrow$ τ reduces $\Rightarrow$ $\underline{\underline{R}}$

(time between interactions smaller)

$$\boxed{R = \rho \frac{l}{A} = \frac{m_e}{e^2 n \tau} \frac{l}{A}}$$

temperature only effect $\Rightarrow$ $R_1 = R_2 = R_3 = R_4 = R_0(1+\alpha\Delta T)$

E_{Th} from ΔT $\Rightarrow$ $E_{Th} = \dfrac{R_1R_4 - R_2R_3}{(R_1+R_2)(R_3+R_4)} = \underline{\underline{0}}$

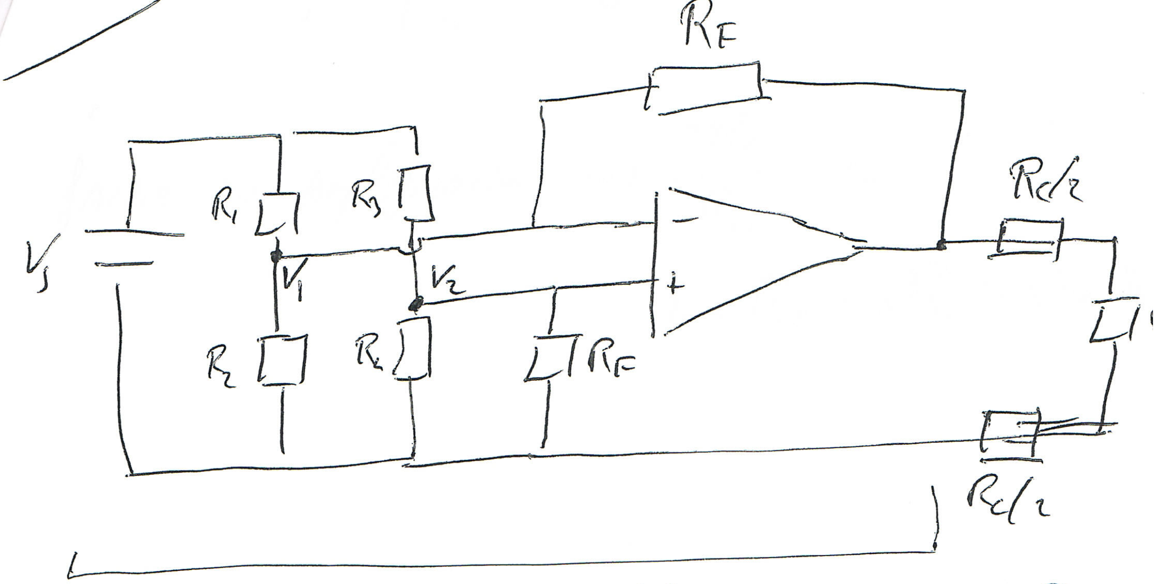
Z_{Th} from ΔT $\Rightarrow$ $Z_{Th} = \dfrac{R_3R_4}{R_3+R_4} + \dfrac{R_1R_2}{R_1+R_2}$

$= R_0(1+\alpha\Delta T) = 105\,\Omega$

$\Rightarrow$ hardly changes $\underline{\underline{V_L}}$!

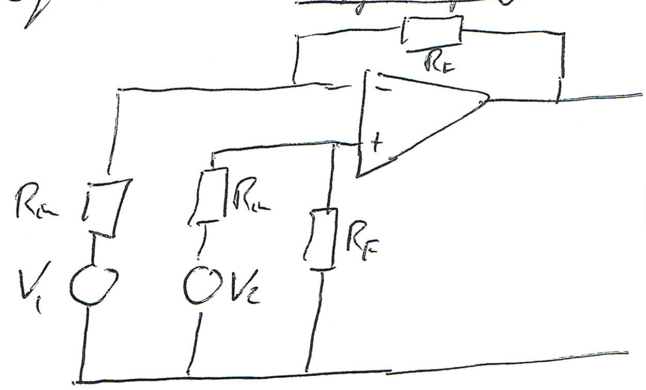
1

5 d



equivalent to diff. amplifier

1



$$V_{out} = (V_2 - V_1)\frac{R_F}{R_{G}}$$

$$R_{in} = \frac{R_1 R_2}{R_1 + R_2} \approx \frac{R_0}{2} \quad [(\delta e)^2 \text{ small!}]$$

$$V_1 = V_s \frac{R_2}{R_1 + R_2} = V_s \frac{R_0(1-\delta e)}{2R_0} = \frac{V_s}{2}(1-\delta e)$$

$$V_2 = V_s \frac{R_4}{R_3 + R_4} = \ldots \frac{(1+\delta e)}{\ldots} = \frac{V_s}{2}(1+\delta e)$$

) cont'd

$$\Rightarrow V_{out} = \frac{2R_F}{R_U} V_s G e$$

factor 10 Amplification, $\Rightarrow \frac{2R_F}{R_U} = 10$

$\hookrightarrow R_F = 5R_U = 500\,\Omega$